Final Technical Report:
AFOSR Grant # F49620-98-1-0256
Aug 1, 1997 - July 31, 2000

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October 1, 2001

1 Executive Summary

The general objective of this research has been the development of the modern mathematical theory of nonlinear dispersive waves, focusing on areas relevant for nonlinear optics. While fundamental theoretical and mathematical behavior is the primary goal of our studies, it must be emphasized that such fundamental understanding is essential to applications of direct importance to the Air Force. Over the three year period of this award the work has concentrated upon optical turbulence and spatio-temporal chaos, dispersive wave turbulence and nonlinear optics – culminating in the initiation of our current study which develops reverse saturable absorbers for laser hardening applications.

In the *area of spatio-temporal* chaos we focused upon turbulent effects in deterministic systems, obtaining two striking results:

1. A very small number (two for our system) of linearly unstable modes is sufficient to trigger the onset of spatial temporal chaos (STC) as characterized by an exponential decay in space of the mutual information;
2. The construction of the effective stochastic dynamics needs only temporal chaos, in contrast to the requirement of full STC as usually believed.

In addition, we introduced the notion of mutual information to capture the transition from temporal to spatio-temporal chaotic states. We also wrote an invited survey summarizing our complete program of temporal chaos in nonlinear Schrodinger-like systems – a program completely carried out under AFOSR support. This survey will appear in the Handbook of Dynamical Systems.

In the *area of dispersive wave turbulence*, we obtained a major result: The identification of a new robust wave spectrum, steeper than predicted by weak turbulence theory. We carried out a detailed, careful and systematic numerical study of dispersive wave turbulence in one spatial dimension – a study which clearly displays four distinct spectra (equi-partition, the direct spectral cascade of weak turbulence theory, the inverse spectral cascade of weak turbulence theory, as well as the new spectra).

These studies also unveil conditions under which the different spectra are observed. Moreover, they contain the most striking verification of the direct cascade of weak turbulence theory to date.

We also investigated the interaction between coherent structures and radiation, the latter being described by dispersive wave turbulence. Understanding this interaction is essential for stochastic prediction of the temporal behavior of coherent structures. The results of this program on dispersive wave turbulence are summarized in the invited survey which appeared in the special millennium issue of the Journal of Mathematical Physics – a research program carried out under AFOSR support.

In the *area of nonlinear optics* our work on coupled bistable switches and optical arrays was written up and published in the Journal of the Optical Society B and in Physica D. Our work on polarization instability for coupled nonlinear Schrodinger equations was written up and published in the Journal of Nonlinear Science. Furthermore in the area of nonlinear optics, a joint project was initiated during this three year period between members of Professor McLaughlin's group at the Courant Institute, Dr. Mary Potasek's group at Brooks AFB and Dr. Ruth Pachter's group at Wright Patterson AFB. This project involves a study of the mathematical properties of reverse saturable absorbers (RSA) for "laser hardening" applications. The work focuses upon propagation effects in nonlinear media containing RSA material such as chromophores. A large scale computer code was developed which incorporates a five level description of the chromophore. This code was then used in an initial study of nonlinear propagation through RSA material.

In addition, to this work of Professor McLaughlin's group, the contract also supported, at various times, Professors Ting, Rosenau, and E.

E and Otto's work on the network formation and visco-elastic response of the polymer network is among the first theoretical study of the non-trivial response in polymer systems during phase separation. Among other things, they found that while shear viscosity is generally destabilizing the polymer network, bulk viscosity plays a stabilizing role. Their numerical results give the first qualitatively correct description for the scaling and morphology of polymer-solvent systems.

Building upon this work, E and Muratov used a kinetic approach to study the phase separation of polymer-liquid crystal systems. They have found that the anisotropic transport in such systems may alter significantly the mechanism for phase separation.

In a joint work with Palffy-Muhoray and Yuan, E has developed a formalism which allows them to obtain, for the first time, explicit analytic expressions for the solutions of Maxwell's equations in general biaxial lossy media. This is significant from the point of view of practical display design applications.

E and Palffy-Muhoray have studied photoalignment of liquid crystals in the presence of dichroic dyes. They found that the torque which reorients the liquid crystal does not come from the transfer of angular momentum from the light, but instead it is the result of an orientational ratchet mechanism, where the dye plays the role of a Brownian motor. This is interesting both from a theoretical and technological point of view. It makes a connection between the interesting transport issues in biological systems and in liquid crystal systems. It also has important applications for display technology (photobuffing of displays), artificial muscle materials (photoelastomers), microactuators and positioners, as well as for optical information storage.

Rosenau's research includes:

(i) Derivation of nonlinear dispersive equations which support compactons and other nonanalytical patterns: It is shown that the motion of curves in the plane is a natural source for such equations (ld+time). He also constructs PDE's which support compact structures on the plane.

(ii) Equations defined by their scale invariance: Many complex processes yield relatively simple patterns and are found either experimentally or numerically to exhibit scale invariance properties over a wide range of scales. He uses this property to determine integral/differential equations which are invariant under the given symmetry. This route enables him to circumvent the inherent difficulty in a direct modelling of such processes. Though scaling symmetry does not define uniquely one equation, but rather a class of equations, he argues, analyzing a number of PDE's having the same scaling, that invariance properties are the crucial feature of a process, and equations sharing the same scaling beget patterns which are quite similar.

(iii) The hallmark of strong dispersion is the possibility to generate nonanalytical structures such as compactons, peakons, spikons etc. These are weak singularities which cause obvious numerical difficulties. He proposes a formulation of these equations (using higher conservation laws) which delays the appearance of singularities, such that all derivatives which enter into the formulation of the problem are smooth. This formulation could have a dramatic impact on the numerical studies of nonlinear dispersive equations.

Ting (with Keller) completed their resolution of the classical problem of the singularity near the line of tangency of an incident wave or reflected wave and a diffracted wave, as first posed by M.J. Lighthill in 1949 when he studied the singularities of the conical solution of a wing in supersonic flow. The analysis was presented in GaMM97 and was published in ZaMM in 1998.

Ting continued his studies in acoustic-structural interaction. An under-

standing of the interaction of panel oscillation with the boundary layer, the outer flow field and incident acoustic waves (simulating jet noise) is essential for the prediction and control of panel fatigue and transmission of external noise through panels of an airframe into the interior. Ting is formulating mathematical models to explain different aspects of the interaction phenomena observed in experiments, and applying asymptotic methods to decouple the acoustic-structural interaction problems as acoustic problems with Neumann boundary conditions and structural oscillations with prescribed loading and a damping term which account for the effect of the acoustic waves induced by the oscillations. The analysis was presented in GaMM98 and was published in ZaMM 1999.

All of work supported under this three year contract has been published in the refereed literature, where the results are described in detail.

2 Technical Report

The technical results are best described from abstracts of selected publications:

2.1 Optical Turbulence and Spatio-temporal Chaos:

- D. CAI, D.W. MCLAUGHLIN AND J. SHATAH, "SPATIAL-TEMPORAL CHAOS AND EFFECTIVE STOCHASTIC DYNAMICS FOR A NEAR INTEGRABLE NONLINEAR SYSTEM", PHYS LETT A **253**, 280-86 (1999).

  We address (i) the onset of spatiotemporal chaos (STC) induced by the hyperbolic structure in a weakly perturbed Nonlinear Schrodinger equation, and (ii) its effective stochastic dynamics (EDS). We obtain the following new results: (1) a very small number (two for our system) of linearly unstable modes is sufficient to trigger the onset of STC as characterized by an exponential decay in space of the mutual information; (2) the construction of the ESD needs only temporal chaos, in contrast to the requirement of full STC as usually believed.

- D. CAI, D.W. MCLAUGHLIN AND K.T.-R MCLAUGHLIN, "THE NONLINEAR SCHRODINGER EQUATION AS BOTH A PDE AND A DYNAMICAL SYSTEM", TO APPEAR HANDBOOK OF DYNAMICAL SYSTEMS (2001)

  Nonlinear dispersive wave equations provide excellent examples of infinite dimensional dynamical systems which possess diverse and fasci-

nating phenomena including solitary waves and wave trains, the generation and propagation of oscillations, the formation of singularities, the persistence of homoclinic orbits, the existence of temporally chaotic waves in deterministic systems, dispersive turbulence and the propagation of spatiotemporal chaos.

Nonlinear dispersive waves occur throughout physical and natural systems whenever dissipation is weak. Important applications include nonlinear optics and long distance communication devices such as transoceanic optical fibers, waves in the atmosphere and the ocean, and turbulence in plasmas. Examples of nonlinear dispersive partial differential equations include the Korteweg de Vries equation, nonlinear Klein Gordon equations, nonlinear Schrodinger equations, and many others.

In this survey article, we choose a class of nonlinear Schrodinger equations (NLS) as prototypal examples, and we use members of this class to illustrate the qualitative phenomena described above. Our viewpoint is one of partial differential equations on the one hand, and infinite dimensional dynamical systems on the other. In particular, we will emphasize global qualitative information about the solutions of these nonlinear partial differential equations which can be obtained with the methods and geometric perspectives of dynamical systems theory.

The article begins with a brief description of the most spectacular success in pde of this dynamical systems viewpoint – the complete understanding of the remarkable properties of the soliton through the realization that certain nonlinear wave equations are completely integrable Hamiltonian systems. This complete integrability follows from a deep connection between certain special nonlinear wave equations (such as the NLS equation with cubic nonlinearity in one spatial dimension) and the linear spectral theory of certain differential operators (the "Zakharov-Shabat" or "Dirac" operator in the NLS case). From this connection the "inverse spectral transform" has been developed and used to represent integrable nonlinear waves. These representations have provided a full solution of the Cauchy initial value problem for several types of boundary conditions, a thorough understanding of the remarkable properties of the soliton, descriptions of quasi-periodic wave trains, and descriptions of the formation and propagation of oscillations as slowly varying nonlinear wavetrains.

In addition, more recent developments are described, including: (i) the formation of singularities and their relationship to dispersive turbulence; (ii) weak turbulence theory; (iii) the persistence of periodic, quasi-periodic, and homoclinic solutions, by methods including normal forms for pde's, Melnikov measurements, and geometric singular perturbation theory; (iv) temporal and spatiotemporal chaos; (v) long-time and small dispersion behavior of integrable waves through Riemann-Hilbert spectral methods. For each topic, the description is necessarily brief; however, references will be selected which should enable the interested reader to obtain more mathematical detail.

2.2 Dispersive Wave Turbulence

- A. MAJDA, D.W. MCLAUGHLIN AND E. TABAK, "A ONE DIMENSIONAL MODEL FOR DISPERSIVE WAVE TURBULENCE", J. NONLINEAR SCIENCE **7**, 9-44 (1997)

  A family of one-dimensional nonlinear dispersive wave equations is introduced as a model for assessing the validity of weak turbulence theory for random waves in an unambiguous and transparent fashion. These models have an explicitly solvable weak turbulence theory which is developed here, with Kolmogorov-type wave number spectra exhibiting interesting dependence on parameters in the equations. These predictions of weak turbulence theory are compared with numerical solutions with damping and driving that exhibit a statistical inertial scaling range over as much as two decades in wave number.

  It is established that the quasi-Gaussian random phase hypothesis of weak turbulence theory is an excellent approximation in the numerical statistical steady state. Nevertheless, the predictions of weak turbulence theory fail and yield a much flatter ($|k|^{-1/3}$) spectrum compared with the steeper ($|k|^{-3/4}$) spectrum observed in the numerical statistical steady state. The reasons for the failure of weak turbulence theory in this context are elucidated here. Finally, an inertial range closure and scaling theory is developed which successfully predicts the inertial range exponents observed in the numerical statistical steady states.

- D. CAI, A. MAJDA, D.W. MCLAUGHLIN AND E. TABAK, "SPECTRAL BIFURCATIONS IN DISPERSIVE WAVE TURBULENCE", PROC. NATL ACAD SCI USA **96**, 14216-14221 (1999)

Dispersive wave turbulence is studied numerically for a class of one-dimensional nonlinear wave equations. Both deterministic and random (white noise in time) forcings are studied. Four distinct stable spectra are observed – the direct and inverse cascades of weak turbulence (WT) theory, thermal equilibrium, and a fourth spectrum (MMT; Majda, McLaughlin, Tabak). Each spectrum can describe long-time behavior, and each can be only metastable (with quite diverse lifetimes) – depending on details of nonlinearity, forcing, and dissipation. Cases of a long-lived MMT transient state decaying to a state with WT spectra, and vice-versa, are displayed. In the case of freely decaying turbulence, without forcing, both cascades of weak turbulence are observed. These WT states constitute the clearest and most striking numerical observations of WT spectra to date –over four decades of energy, and three decades of spatial, scales. Numerical experiments that study details of the composition, coexistence, and transition between spectra are then discussed, including: (i) for deterministic forcing, sharp distinctions between focusing and defocusing nonlinearities, including the role of long wavelength instabilities, localized coherent structures, and chaotic behavior; (ii) the role of energy growth in time to monitor the selection of MMT or WT spectra; (iii) a second manifestation of the MMT spectrum as it describes a self-similar evolution of the wave, without temporal averaging; (iv) coherent structures and the evolution of the direct and inverse cascades; and (v) nonlocality (in k-space) in the transferral process.

- D. CAI AND D.W. MCLAUGHLIN, "CHAOTIC AND TURBULENT BEHAVIOR OF UNSTABLE ONE-DIMENSIONAL NONLINEAR DISPERSIVE WAVES", JOURNAL OF MATHEMATICAL PHYSICS, **41**, NO 6, 4125-4153 (2000)

  In this article we use one-dimensional nonlinear Schroedinger equations (NLS) to illustrate chaotic and turbulent behavior of nonlinear dispersive waves. It begins with a brief summary of properties of NLS with focusing and defocusing nonlinearities. In this summary we stress the role of the modulational instability in the formation of solitary waves and homoclinic orbits, and in the generation of temporal chaos and of spatiotemporal chaos for the nonlinear waves. Dispersive wave turbulence for a class of one-dimensional NLS equations is then described in detail – emphasizing distinctions between focusing and defocusing cases, the role of spatially localized, coherent structures, and

their interaction with resonant waves in setting up the cycles of energy transfer in dispersive wave turbulence through direct and inverse cascades. In the article we underline that these simple NLS models provide precise and demanding tests for the closure theories of dispersive wave turbulence. In the conclusion we emphasize the importance of effective stochastic representations for the prediction of transport and other macroscopic behavior in such deterministic chaotic nonlinear wave systems.

- D. CAI, A. MAJDA, D. MCLAUGHLIN AND E. TABAK, "DISPERSIVE WAVE TURBULENCE IN ONE DIMENSION", PHYSICA D, **152-53**, 551-572 (2001)

  In this article, we study numerically a one dimensional model of dispersive wave turbulence. The article begins with a description of the model which we introduced earlier, followed by a concise summary of our previous results about it. In those previous studies, in addition to the spectra of weak turbulence (WT) theory, we also observed another distinct spectrum (the "MMT-spectrum"). Our new results, presented here, include: (i) A detailed description of coexistence of spectra at distinct spatial scales, and the transitions between them at different temporal scales; (ii) The existence of a stable MMT front in k space which separates the WT cascades from the dissipation range, for various forms of strong damping including "selective dissipation"; (iii) The existence of turbulent cycles in the one dimensional model with focusing nonlinearity, induced by the interaction of spatially localized coherent structures with the resonant quartets of dispersive wave radiation; (iv) The detailed composition of these turbulent cycles — including the self-similar formation of focusing events (distinct in the forced and freely decaying cases), and the transport by the WT direct and inverse cascades of excitations between spatial scales. This one dimensional model admits a very precise and detailed realization of these turbulent cycles and their components. Our numerical experiments demonstrate that a complete theory of dispersive wave turbulence will require a full description of the turbulent field over all spatial scales (including those of the forcing and dissipation), and over extremely long times (as the nonlinear turnover time becomes very long in the weakly nonlinear limit). And, in the focusing case, a complete theory must also incorporate the interaction of localized coherent structures with resonant radiation.

2.3 Nonlinear Optics I: Coupled Bistable Switches

- Y. Chen and D.W. McLaughlin, "Focusing-Defocusing Effects for Diffusion Dominated Bistable Optical Arrays", Journal of the Optical Society B, **16**, No. 7, 1087-1098 (1999)

  Bistable responses of Fabry-Perot cavities and optical arrays in the presence of diffraction and diffusion are studied both analytically and numerically. The model is a pair of nonlinear Schrodinger equations coupled through a diffusion equation. The numerical computations are based on a split-step method, with three distinct characteristics. In these diffusion-dominated arrays with weak diffraction, this study demonstrates that focusing nonlinearity can improve the response characteristics significantly. The primary results of the study are that (1) for diffusion-dominated media a small amount of diffraction is sufficient to alter optical bistability significantly; (2) focusing nonlinearities enhance optical bistability in comparison with defocusing nonlinearities; (3) in diffusion-dominated media these focusing-defocusing effects are quite distinct from self-focusing behavior in Kerr media; (4) in the presence of diffraction the response of the array can be described analytically by a reduced map, whose derivation can be viewed as an extension of Firth's diffusive model to include weak diffraction; (5) this map is used to explain analytically certain qualitative features of bistability, as observed in the numerical experiments; and (6) optimal spacing predictions are made with a reduced map and verified with numerical simulations of small all-optical arrays.

2.4 Nonlinear Optics II: Polarization Effects

- M.G. Forest, D.W. McLaughlin, D. Muraki and O. Wright, "Non-Focusing Instabilities in Coupled, Integrable Nonlinear Schroedinger PDE's", Journal of Nonlinear Science **10**, 291-331 (2000)

  The nonlinear coupling of two scalar nonlinear Schrodinger (NLS) fields results in nonfocusing instabilities that exist independently of the well-known modulational instability of the focusing NLS equation. The focusing versus defocusing behavior of scalar NLS fields is a well-known model for the corresponding behavior of pulse transmission in optical fibers in the anomalous (focusing) versus normal (defocus-

ing) dispersion regime. For fibers with birefringence (induced by an asymmetry in the cross section), the scalar NLS fields for two orthogonalpolarization modes couple nonlinearly. Experiments by Rothenberg have demonstrated a new type of modulational instability in a birefringent normal dispersion fiber, and he proposes this cross-phase coupling instability as a mechanism for the generation of ultrafast, terahertz optical oscillations. In this paper the nonfocusing plane wave instability in an integrable coupled nonlinear Schrodinger (CNLS) partial differential equation system is contrasted with the focusing instability from two perspectives: traditional linearized stability analysis and integrable methods based on periodic inverse spectral theory. The latter approach is a crucial first step toward a nonlinear, nonlocal understanding of this new optical instability analogous to that developed for the focusing modulational instability of the sine-Gordon equations by Ercolani, Forest and McLaughlin and the scalar NLS equation by Tracy, Chen and Lee, Forest and Lee, and McLaughlin, Li, and Overman.

2.5 Nonlinear Optics III: Reverse Saturable Absorbers in Laser Hardening

- S. Kim D.W. McLaughlin and M. Potasek, "Propagation of the Electromagnetic Field in Optical Limiting Reverse Saturable Absorbers", Phys Rev A, **61**, 025801-1 - 025801-4 (2000)

  Reverse saturable absorbers are of considerable interest for optical limiting. Using the electric dipole perturbation, we derived the rate equation for a five-level system describing reverse saturable absorbers. Traditional theories for the propagating laser beam in these materials are expressed in terms of the optical intensity. However, with the introduction of high power short pulsed lasers, the propagation of light in these materials may be subject to nonlinear phenomena such as self-focusing and self-phase modulation. Furthermore, conventional theories treat the laser light as a continuous wave or as a very broad temporal pulse in which dispersive effects are neglected. In order to incorporate these other nonlinear/dispersive effects, and therefore determine their influence in reverse saturable absorbers, we derived an equation for the propagation of the electromagnetic field, rather than the intensity, coupled to the rate equations for a five-level system. We

also coupled our theory to experimentally measurable parameters for these materials and detailed the various physical approximations used to obtain the rate equations.

- M.J. POTASEK, S. KIM, AND D.W. MCLAUGHLIN, "ALL-OPTICAL POWER LIMITING", J. NONLINEAR OPT. PHYS MAT, **9**, 343-364 (2000)

  We derived a numerical technique for the propagation of the electromagnetic field in a five-level reverse saturable absorber including the nonlinear Kerr effect and dispersion. The numerical method combines the split step beam propagation method and the Crank-Nicholson method. Using our numerical technique we observed new behavior, not previously observed nor predicted to our knowledge, including the temporal splitting caused by the dynamics of the carrier densities in a reverse saturable absorber and the enhancement of absorption due to the Kerr nonlinearity. Our numerical calculation enables the prediction of nonlinear absorption using material parameters such as the absorption cross-sections and decay rates. We can also investigate the interplay between the optical pulse properties such as the temporal pulse width, spatial radius, incident energy and the carrier dynamics and nonlinear absorption of the reverse saturable absorber.

3 Personnel Associated with Research Effort

- FACULTY

  David W. McLaughlin;
  Weinan E;
  Lu Ting

- POST-DOCS

  Cyrill Muratov;
  Sukkeun Kim

  In addition, while not receiving explicit support from the grant, David Cai and Jim Wielaard have been involved in parts of the research effort.

- GRADUATE STUDENTS

Roy Goodman (Graduate Student, working with McLaughlin);

- OTHER

  Analisa Calini (Assist. Prof, Charleston College);
  Constance Schober (Assist Prof, Old Dominion University);
  Ruth Pachter (Research Scientist, Wright Patterson AFB);
  Mary Potasek (Research Scientist, Brooks AFB);
  Philip Rosenau (Prof, Tel Aviv University)]

- ADDITIONAL:

  Y. Chen (former PhD student, not supported on this grant);
  G. Forest (Professor, Univ of NC, not supported on this grant);
  A. Majda (Prof, Courant Inst, not supported by AFOSR);
  D. Muraki (Assist Professor, Courant Inst, not supported AFOSR);
  K. T-R McLaughlin (Assist Prof, Univ Ariz, not supported AFOSR);
  J. Shatah (Prof, Courant Inst, not supported by AFOSR);
  E. Tabak (Prof, Courant Inst, not supported by AFOSR);
  O. Wright (Instructor, Univ of NC, not supported on this grant).

Publications

Submitted and in Preparation

1. D. Cai, D.W. McLaughlin and J. Shatah, “Spatiotemporal Chaos in Spatially Extended Systems”, J. Mathematics and Computers in Simulation, **55**, 329-340 (2001);

2. D. Cai, A.J. Majda, D.W. McLaughlin and E. Tabak, “Dispersive Wave Turbulence In One Dimension”, Physica D, 152-153, 551-572 (2001);

3. M.J. Potasek, S. Kim, D.W. McLaughlin, “Optical Power Limiting in Organo-Metallic Materials”, accepted to the conference on Power Limiting Materials and Devices, SPIE’s 45th Annual Meeting, International Symposium on Optical Science and Technology, San Diego, CA, 30 - July - 4 August, 2000, paper to be published in conference proceedings.

Accepted

Journals

1. David W. McLaughlin and Y. Li, "Homoclinic Orbits and Chaos in Discretized Perturbed NLS Systems - Part I. Homoclinic Orbits", J. Nonlinear Science, **7**, 211-269 (1997);
2. A. Majda, D.W. McLaughlin and E.Tabak, "A One Dimensional Model for Dispersive Wave Turbulence", J. Nonlinear Science **7**, 9-44 (1997);
3. S. Jin, D. Levermore and D.W. McLaughlin, "The Semiclassical Limit of the Defocusing NLS Hierarchy", Comm. Pure and Appl Math (1999);
4. D. Cai, D.W. McLaughlin and J. Shatah, "Spatial-Temporal Chaos and Effective Stochastic Dynamics for a Near Integrable Nonlinear System", Phys Lett A **253**, 280-86 (1999);
5. Y. Chen and D.W. McLaughlin, "Focusing-Defocusing Effects for Diffusion Dominated Bistable Optical Arrays", Journal of the Optical Society B, **16**, No. 7, 1087-1098 (1999);
6. Y. Chen and D.W. McLaughlin, "Diffraction Effects on Diffusive Bistable Optical Arrays", Physica D **138**, 163-195 (2000);
7. D. Cai, D.W. McLaughlin and K.T-R McLaughlin, "The Nonlinear Schrodinger Equation as both a PDE and a Dynamical System", to appear Handbook of Dynamical Systems (2000);
8. D. Cai, A. Majda, D.W. McLaughlin and E. Tabak, "Spectral Bifurcations in Dispersive Wave Turbulence", Proc. Natl Acad. Sci USA **96**, 14216-14221 (1999);
9. D. Cai and D.W. McLaughlin, "Chaotic and Turbulent Behavior of Unstable One-Dimensional Nonlinear Dispersive Waves", Journal of Mathematical Physics, **41**, No. 6, 4125-4153, (2000);
10. M.G. Forest, D.W. McLaughlin, D. Muraki and O. Wright, "Non-Focusing Instabilities in Coupled, Integrable Nonlinear Schrodinger PDE's", Journal of Nonlinear Science **10**, 291-331 (2000);
11. M.J. Potasek, S. Kim, and D.W. McLaughlin, "All-Optical Power Limiting", J. Nonlinear Opt. Phys & Mat, **9**, 343-364 (2000);

12. M.J. Potasek and S. Kim, "Time Delayed Nonlinear Absorption Effects on Spatiotemporal Propagation for Femtosecond Pulses", 18th Congress of the International Commission for Optics (ICO XVIII), San Francisco,CA, (2-6 Aug 1999);

13. S. Kim, D.W. McLaughlin and M. Potasek, "Propagation of the Electromagnetic Field in Optical Limiting Reverse Saturable Absorbers", Phys Rev A, **61**, 025801-1 - 025801-4 (2000);

14. L. Ting and J.B. Keller, "Weak Shock Diffraction and Singular Rays", an extended abstract, ZaMM, 1998;

15. L. Ting, "Decoupling Acoustic Structural Waves", presented at GaMM98, Session 13.5, University Bremen, April 8, 1998, ZaMM, 1999;

16. L. Ting, R. Klein and O. Knio, "Asymptotic Theory of Slender Vortex Filaments - Old and New", Proceedings of the IUTAM Symposium on Dynamics of Slender Vortices, RWTH Aachen, August 31- Sept 3, 1997, Kluwer Academic Publ, Dordrecht, 3-20, 1998;

17. O. Knio and L. Ting, "Noise Emission due to Slender Vortices - Solid Body Interaction", Proceedings of the IUTAM Symposium on Dynamics of Slender Vortices, RWTH Aachen,September 1-3, 1997; 369-378, Kluwer Publ., Dordrecht, 1998;

18. M.J. Miksis and L.Ting, "Structural Acoustic Interactions and On Surface Conditions:, IMA Volumes in Mathematics and its Applications, 96, "Computational Wave Propagation", Editors, B. Engquist and G.A. Kriegsmann, 165-178, Springer-Verlag, (1997);

19. O.M. Knio and L. Ting, "Vortical Flow outside a Sphere and Sound Generation", SIAM J. Appl. Math. Vol 57, No. 4, 972-981, (1997);

20. O. Knio, R. Klein and L. Ting., "Interaction of a Slender Vortex Filament with a Rigid Sphere: Dynamics and Far-Field Noise", J. Acoust Soc Amer., 108, 83-98 (1998);

21. Weinan E, "Nonlinear Continuum Theory of Smectic A Liquid Crystals", Arch. Rat. Mach. Analy., (1997);

22. Weinan E and P. Palffy-Muhoray, "Phase Separation of Incompressible systems;, Phys Rev E, Rapid Communications, (1997);

23. Weinan E and P. Palffy-Muhoray, "Wavelength Selection in Slowly Quenched Systems", Molecular Crystal and Liquid Crystals, 292 - 345, (1997);

24. Weinan E and F. Otto, "Thermodynamically Driven Incompressible Fluid Mixtures", J. Chem. Phys. 107 (23), 10177-10184, (1997);

25. Weinan E, "Nonlinear Continuum Theory of Smectic A Liquid Crystals", Arch. Rat. Mech. Anal., 137 159-175 (1997).

Conferences

1. D. McLaughlin, "Homoclinic and Chaotic Behavior in Nonlinear PDE's", Invited Lecture, IMA Conference on Dynamics (Oct 1997);

2. D. McLaughlin and J. Shatah, "Geometric Singular Perturbation Theory and Homoclinic Behavior in PDS's", Invited Short course, AMS-MEXICO Meeting (Dec 1997);

3. D. McLaughlin, Invitation to speak at the ICIAM 99, Edinburgh, June, 1999 (Cancelled);

4. D. Cai, "Large time spatiotemporal statistical properties of dispersive waves;, Invited Lecture at Workshop on Hamiltonian Mechanics and Small Divisors in PDEs, ICIAM, Edinburgh, Scotland, June, 1999;

5. D. Cai, "Spatiotemporal chaos and effective stochastic dynamics;, International Conference of Applied Mathematics and Computational Sciences on Nonlinear Evolution Equations and Wave Phenomena, Athens, Georgia, April 1999;

6. D. McLaughlin, "Spatial-temporal chaos in NLS Equations", Invited Lecture at Third Workshop on Nonlinear Dynamics: Chaos, Transport and Transition to Turbulence;

7. D. McLaughlin, "Spatial-temporal chaos in NLS Equations", Invited Lecture at Applied Math Seminar, University of Arizona (October 1998);

8. D. McLaughlin, "Spectral Bifurcations in Dispersive Wave Turbulence", Invited Lecture, University of Hong Kong (March 2000);

9. D. McLaughlin, "Dispersive Wave Turbulence", Invited Lecture, IMA Workshop on Dispersive Corrections to Transport Equations, (May, 2000);
10. D. McLaughlin, "Spectral Bifurcations in Dispersive Wave Turbulence" Invited Lecture, 2000 AMS-IMS-SIAM Summer Research Conference on Dispersive Wave Turbulence (June 2000);
11. D. McLaughlin, "Spectral Bifurcations in Dispersive Wave Turbulence", Invited Lecture, Rome, Italy, (September 6-9, 2000);
12. MJ Potasek and S. Kim, "Time Delayed Nonlinear Absorption Effects on Spatiotemporal Propagation for Femtosecond Pulses", 18th Congress of International Committee for Optics (ICO XVIII) (Aug, 1999).
13. Weinan E, "Phase Separation in Polymer Systems:, Invited Lecture, Workshop in Nonequilibrium Dynamics, Los Alamos, (April 1998);
14. Weinan E, "Exact Results on PDEs for Burgers Equation", Invited Lecture, Turbulence in the 21 Century, Los Alamos, (May 1998);
15. Weinan E, "Pattern Formation in Liquid Crystal Phase Transition", Invited Lecture, IMA Workshop on Continuum Mechanics and Nonlinear PDEs, (June, 1998);
16. Weinan E, "Invited plenary speaker, International Congress of Chinese Mathematicians, Beijing, Dec 1998 (was not able to attend);
17. Weinan E, "Stochastic PDEs in turbulence theory", Conference on Stochastic Evolution Equations, Los Alamos, (Sept 1999);
18. Weinan E, "Stochastic PDEs in turbulence theory", Nordic conference on nonequilibrium statistical physics, Denmark, (Sept 1999);
19. C. Muratov, "Theory of phase separation kinetics in polymer-liquid crystal systems", AMS regional meeting, Penn State University, (Oct 1998);
20. C. Muratov, "Theory of phase separation kinetics in polymer-liquid crystal systems", SIAM Meeting on Dynamical Systems, Snowbird (May 1999).

Interactions/Transitions

Participation/Presentations At Meetings, Conferences, Seminars, Etc

E: Co-Organizer: Minisymposium in Dynamics and Defects in Polymer-Liquid Crystal Systems, SIAM Meeting on Material Sciences, Philadelphia, (May 1997);
E: Co-Organizer: CIMS/ALCOM/NIST workshop on phase separation polymer-liquid crystal systems, April 25-26, New York.

Consultative and Advisory Functions to Other Laboratories and Agencies

McLaughlin:
Advisory Function, Brooks AFB, Mary Potasek, "Advice of short pulse numerical code";

Cooperation, Wright Patterson and Brooks AFB, Ruth Pachter and Mary Potasek, "Initiating Project on light-matter interactions";

Member, Review Panel, NSF Science and Technology Center Preproposals;
Member, Advisory Committee Research Strategy Planning of Mathematical and Computer Science Division of Army Research Office;

E:
Member, Review Panel, NSF Proposals in Applied Mathematics

Transitions

McLaughlin's collaboration with members of three laboratories (Courant Institute, Brooks AFB, Wright Patterson AFB) continues.

New Discoveries, Inventions or Patent Disclosures

Honors/Awards

McLaughlin

1. 1994: Director, Courant Institute of Mathematical Sciences, New York University;
2. 2000: Fellow of the American Academy of Arts and Sciences

REPORT DOCUMENTATION PAGE

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AFRL-SR-BL-TR-01-
0536

1. REPORT DATE *(DD-MM-YYYY)*
October 1, 2001

Final Technical Report

3. DATES COVERED *(From - To)*
Aug 1, 1997 - July 31, 2000

4. TITLE AND SUBTITLE
Mathematical Nonlinear Optics

5a. CONTRACT NUMBER

5b. GRANT NUMBER
F49620-98-1-0256

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
David W. McLaughlin

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

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8. PERFORMING ORGANIZATION REPORT NUMBER

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Air Force Office of
Scientific Research
801 North Randolph - Rm. 732
Arlington, VA 22203-1977

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFOSR)
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HAS BEEN REVIEWED AND IS APPROVED FOR PUBLIC RELEASE
LAW AFR 190-12. DISTRIBUTION IS UNLIMITED.

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release,
distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

This research concerns the development of the modern mathematical theory of nonlinear dispersive waves focusing on areas relevant for nonlinear optics and developing fundamental understanding which is essential to applications of direct importance to the Air Force.

The work has concentrated upon optical turbulence and spatio-temporal chaos, dispersive wave turbulence, and nonlinear optics - culminating in the initiation of a study of reverse saturable absorbers for laser hardening applications. The major findings include
(i) the onset of spatio-temporal chaos occurs with only two instabilities; (ii) a new spectra observed for dispersive wave turbulence; (iii) the performance of coupled bistable switches and optical arrays; and (iv) a classification of polarization instability for coupled nonlinear Schrodinger equations.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:
a. REPORT unclassified
b. ABSTRACT unclassified
c. THIS PAGE unclassified

17. LIMITATION OF ABSTRACT

18. NUMBER OF PAGES

19a. NAME OF RESPONSIBLE PERSON
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Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18